5.MD Box of Clay

Alignments to Content Standards

- Alignment: 5.MD.C

Tags

Tags: AMC

A box 2 centimeters high, 3 centimeters wide, and 5 centimeters long can hold 40 grams of clay. A second box has twice the height, three times the width, and the same length as the first box. How many grams of clay can it hold?

Commentary

This task provides an opportunity to compare the relative volumes of boxes in order to calculate the mass of clay required to fill them. These relative volumes can be calculated geometrically, filling the larger box with smaller boxes, or arithmetically using the given dimensions.

This purpose of this task is to help students understand what happens when you scale the dimensions of a right rectangular solid. At some point students need to understand that if you (for example) double the length, width, and height of a rectangular solid, then the volume increases by a factor of $2 \times 2 \times 2 = 8$. Before they get to the point of generalizing this phenomenon, they should think about the effects of scaling the different dimensions by different factors, as they do in this task.

This is a high-level instructional task for fifth graders, but it fits squarely within the scope of work that students are doing to understand the volume of right rectangular solids with whole-number side-lengths in fifth grade. Depending on how comfortable students are with visualizing rectangular solids, they might benefit from having snap-cubes or some other physical model to help them solve this problem.

This task was adapted from problem #3 on the 2012 American Mathematics Competition (AMC) 12A Test. The responses to the multiple choice answers for the problem had the following distribution:

| Choice	Answer	Percentage of Answers
(A)	120	2.08
(B)	160	3.60
(C)	200	5.15
(D)*	240	83.82
(E)	280	0.99
Omit	--	4.33

Of the 72,238 students who participated, 28,268 or 39% were in 12th grade, 34,124 or 47% were in 11th grade, 4,615 or 6% were in 10th grade, and the remainder were below 10th grade.
To see an annotated version of this and other Illustrative Mathematics tasks as well as other Common Core aligned resources, visit Achieve the Core.

Solutions

Solution: 1 Geometric visualization

The second box has 3 times the width and the same length as the first, smaller box. So we can fit three of the smaller boxes inside the second box to make one layer which will be 2 cm high. The second box is 2 times as high as the smaller one so we can add one more layer of three smaller boxes to fill the second box.
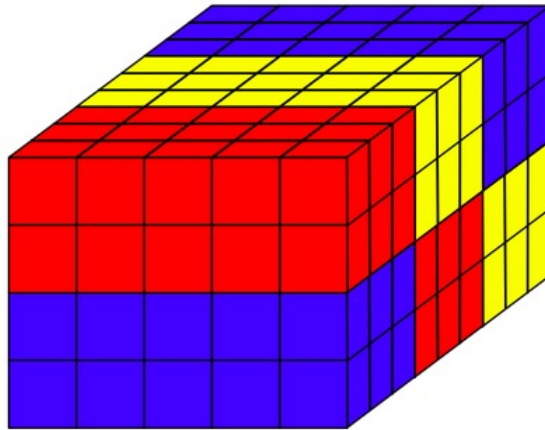
This means that it takes 6 small boxes to fill the large box so the large box holds six times as much as the small box. Since the small box holds 40 grams of clay, the large box holds $6 \times 40 = 240$ grams fo clay.

Solution: 2 Arithmetic comparison of volumes

The first box is 2 centimeters high, 3 centimeters wide, and 5 centimeters long so it has volume

$$2\text{cm} \times 3\text{cm} \times 5\text{cm} = 30 \text{ cubic centimeters}$$

and it holds 40 grams of clay. The second box is 4 centimeters high, 9 centimeters wide, and 5 centimeters long so its volume is

$$4\text{cm} \times 9\text{cm} \times 5\text{cm} = 180 \text{ cubic centimeters.}$$

Since the volume of the second box is $180 \div 30 = 6$ times bigger, it can hold 6 times as much clay. So the second box can hold $6 \times 40 = 240$ grams of clay.